ME 204 Engineering Mechanics: Dynamics

Dynamics of Particles – Plane Kinematics of Rigid Bodies

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Kinematics

- Study of geometry of motion
- Used to relate
  - displacement
  - velocity
  - acceleration
without reference to the cause of the motion (Force not considered)
Kinetics

- Study of relation between
  - the forces acting on the body
  - the mass of the body
  - the motion of the body

- Used
  - to predict the motion caused by the forces or
  - to determine the forces required to produce a motion
Plane Kinematics of Rigid Bodies

Rigid Body
System of particles where the distances between particles are constant.

Plane Motion
All parts of the body move in parallel planes.
Types of Rigid Body Plane Motion

- Translation
  - Rectilinear Translation
  - Curvilinear Translation
- Fixed Axis Rotation
- General Plane Motion
Make use of geometric relations defining the configuration of the body.

Take time derivatives of the defining geometric relations to obtain velocity and acceleration.

If the geometry is complex, use relative motion analysis.
Absolutely Motion

Rolling Wheel (no slip)

\[ s = r \theta \]

\[ v_o = s = r \dot{\theta} = r \omega \]

\[ a_o = v_o = s = r \theta = r \omega = r \alpha \]
Absolute Motion

Rolling Wheel (no slip)

\[
x = s - r \sin \theta = r(\theta - \sin \theta)
\]
\[
x = r \dot{\theta}(1 - \cos \theta) = v_o (1 - \cos \theta)
\]
\[
x = v_o (1 - \cos \theta) + v_o \dot{\theta} \sin \theta
\]
\[
x = a_o (1 - \cos \theta) + r \omega^2 \sin \theta
\]
\[
y = r - r \cos \theta = r(1 - \cos \theta)
\]
\[
y = r \dot{\theta} \sin \theta = v_o \sin \theta
\]
\[
y = v_o \sin \theta + v_o \dot{\theta} \cos \theta
\]
\[
y = a_o \sin \theta + r \omega^2 \cos \theta
\]

For \( \theta = 0 \)
\[
x = 0
\]
\[
y = r \omega^2
\]

For any \( \theta \)
\[
\ddot{v} = x \ddot{i} + y \ddot{j}
\]
\[
\ddot{a} = x \ddot{i} + y \ddot{j}
\]
Rotation of the lever $OA$ is controlled by the motion of the contacting circular disk whose center is given a horizontal velocity $v$. Determine the expression for the angular velocity $\omega$ of the lever $OA$ in terms of $x$. 
One of the most common mechanisms is the slider-crank. Express the angular velocity $\omega_{AB}$ and angular acceleration $\alpha_{AB}$ of the connecting rod $AB$ in terms of the crank angle $\theta$ for a given constant crank speed $\omega_0$. Take $\omega_{AB}$ and $\alpha_{AB}$ to be positive counterclockwise.

In the diagram:
- $\theta$ is the crank angle.
- $\omega_0$ is the constant crank speed.
- $l$ and $r$ are the lengths of the connecting rod and crank, respectively.

At point $B$, there is a rotation.
Relative Velocity

Relative Velocity Due to Rotation

\[ \Delta \vec{r}_A = \Delta \vec{r}_B + \Delta \vec{r}_{A/B} \]

\[ \vec{v}_A = \vec{v}_B + \vec{v}_{A/B} \]

\[ \vec{v}_{A/B} = \rho \omega \]

\[ \vec{v}_{A/B} = \omega \times \vec{r} \]
Relative Velocity

**Interpretation of Relative-Velocity Equation**

Velocity of $A = \text{vector sum of translational portion } \vec{v}_B + \text{ rotational portion } \vec{v}_{A/B} = \vec{\omega} \times \vec{r}$
The circular disk rolls without slipping with a counterclockwise angular velocity $\omega = 4 \text{ rad/s}$. For the instant represented, write the vector expressions for the velocity of $A$ with respect to $B$ and for the velocity of $P$. 

![Diagram of a circular disk with labeled points A, B, P, and C. The disk rolls on a flat surface with the radius of the disk marked as 200 mm and the diameter marked as 300 mm. The disk's angular velocity is indicated as $\omega = 4 \text{ rad/s}$. The point O is the center of the disk. The coordinate axes x and y are shown.]}
End A of the link has a downward velocity $v_A$ of 2 m/s during an interval of its motion. For the position where $\theta = 30^\circ$ determine the angular velocity $\omega$ of AB and the velocity $v_G$ of the midpoint G of the link. Solve the relative-velocity equations, first, using the geometry of the vector polygon and, second, using vector algebra.
The elements of the mechanism for deployment of a spacecraft magnetometer boom are shown. Determine the angular velocity of the boom when the driving link $OB$ crosses the $y$-axis with an angular velocity $\omega_{OB} = 0.5 \text{ rad/s}$ if $\tan \theta = 4/3$ at this instant.
Instantaneous Center of Zero Velocity

**Locating the Instantaneous Center**

**Instantaneous center of zero velocity (C)**: For a body with plane motion, the unique point on the body such that, instantaneously, \( \vec{v}_c = 0 \).

**Instantaneous axis of zero velocity**: The axis which is normal to the plane of motion and passes through C. As far as the velocities are concerned, the body can be considered to be executing pure rotation, instantaneously, around the instantaneous axis of zero velocity.
Locating the Instantaneous Center

\[ v_A = \omega r_A \]
\[ v_B = \omega r_B \]

- \( \vec{\omega} \) and C can be determined if \( \vec{v}_A \) and direction of \( \vec{v}_B \) (or, \( \vec{v}_B \) and direction of \( \vec{v}_A \)) known.
- If \( \vec{\omega} \) and C known, velocity of any point can be determined.
Motion of the Instantaneous Center

The position of C changes as the position of the body changes.

**Space centrode**: Locus of C on the fixed (i.e., nonmoving) body.

**Body centrode**: Locus of C on the body itself.

Body centrode rolls, without slippage, on the space centrode during motion (C is contact point of two curves).
Motion of the bar is controlled by the constrained paths of A and B. If the angular velocity of the bar is $2 \text{ rad/s}$ counterclockwise as the position $\theta = 45^\circ$ is passed, determine the speeds of points A and P.
The hydraulic cylinder produces a limited horizontal motion of point $A$. If $v_A = 4 \text{ m/s}$ when $\theta = 45^\circ$, determine the magnitude of the velocity of $D$ and the angular velocity $\omega$ of $ABD$ for this position.
Motion of the roller $A$ against its restraining spring is controlled by the downward motion of the plunger $E$. For an interval of motion the velocity of $E$ is $v = 0.2 \text{ m/s}$. Determine the velocity of $A$ when $\theta$ becomes $90^\circ$. 
Homework 4 (Chapter 5)

Problems for Week 09

- **Problem 5/53**
  - There will be additional problems for the next week(s).

- **Problem 5/56**
  - Solve each week's problems in that week, do not wait for the last day!

- **Problem 5/75**
  - Submit the Homework 04 in due time.

- **Problem 5/84**
  - You will have a quiz in the 1st hour of Week 11. A problem will be asked from the Homework 4 questions.

- **Problem 5/108**
  - Your homework grade will be assessed only from that question of your homework.

- **Problem 5/120**

**Due date**: 1st hour of Week 11
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