ME 204 Engineering Mechanics: Dynamics

Dynamics of Particles –
Plane Kinematics of Rigid Bodies

Asst.Prof.Dr.Turgut AKYÜREK

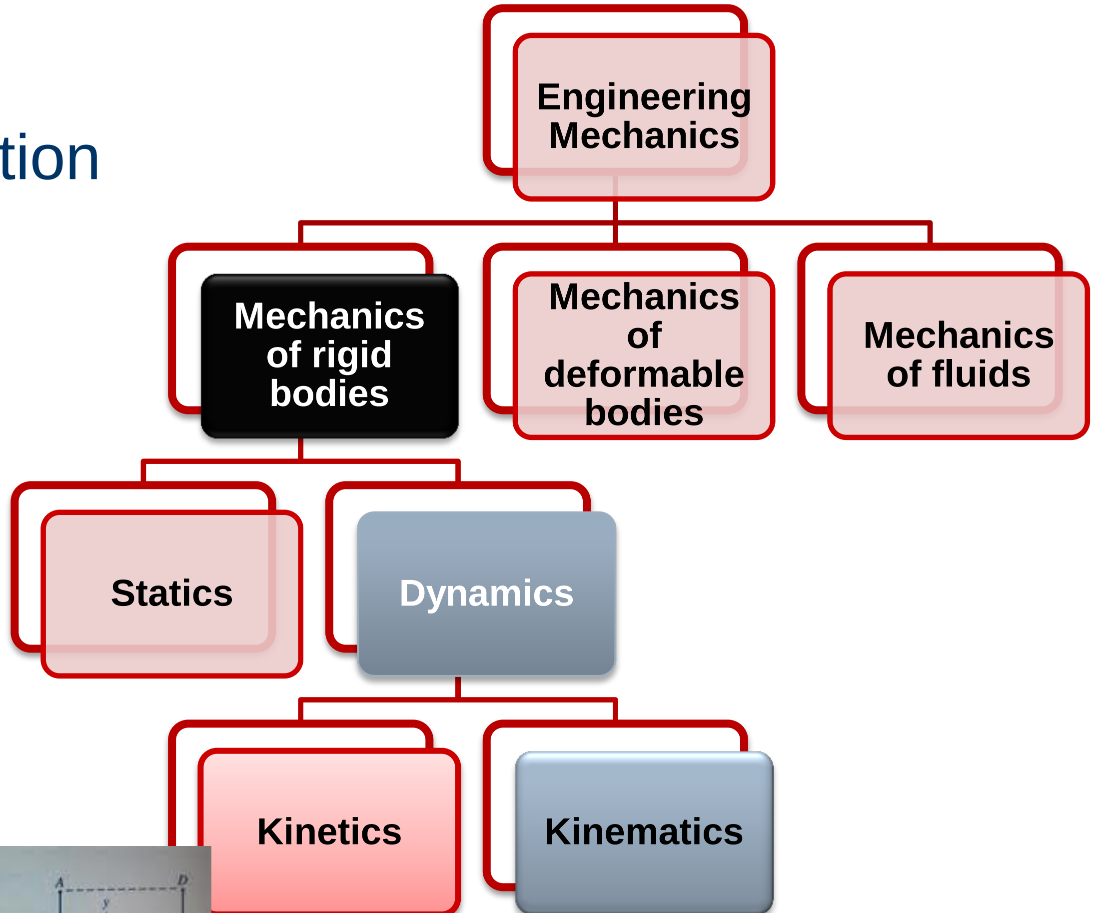
Çankaya University, Ankara

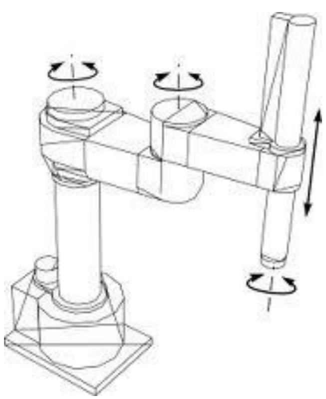
Rigid Body Mechanics

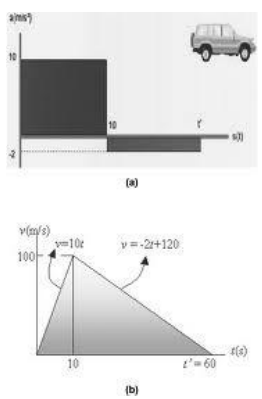
Kinematics

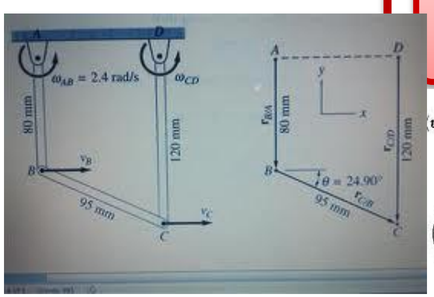
- Study of geometry of motion
- Used to relate
  - displacement
  - velocity
  - acceleration

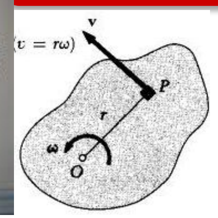
  without reference to the cause of the motion (Force not considered)







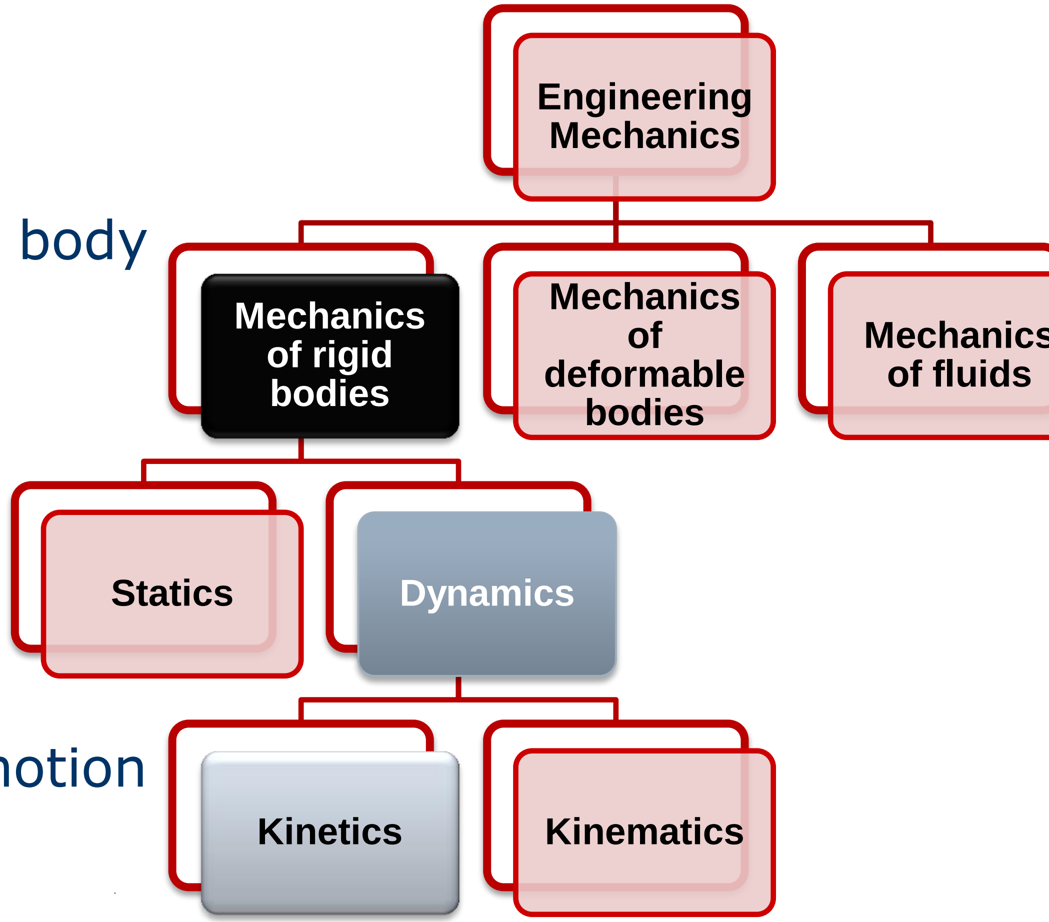


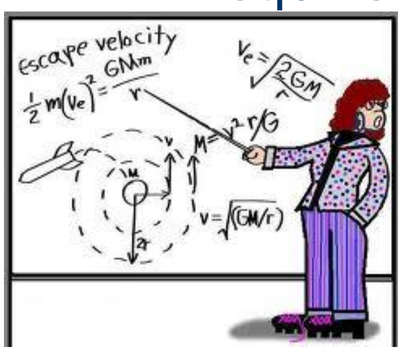


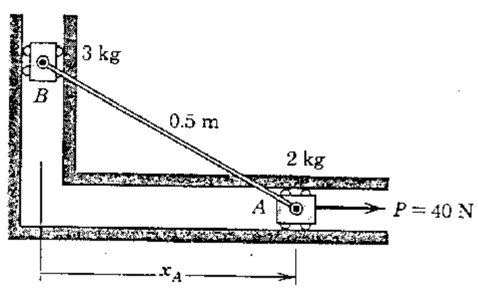
Rigid Body Mechanics

Kinetics

- Study of relation between
  - the forces acting on the body
  - the mass of the body
  - the motion of the body
- Used
  - to predict the motion caused by the forces or
  - to determine the forces required to produce a motion







Rigid Body

System of particles where the distances between particles are constant.

Plane Motion

All parts of the body move in parallel planes.

Types of Rigid Body Plane Motion

- Translation
  - Rectilinear Translation
  - Curvilinear Translation
- Fixed Axis Rotation
- General Plane Motion

Absolute Motion

- Make use of geometric relations defining the configuration of the body.
- Take time derivatives of the defining geometric relations to obtain velocity and acceleration.
- If the geometry is complex, use relative motion analysis.

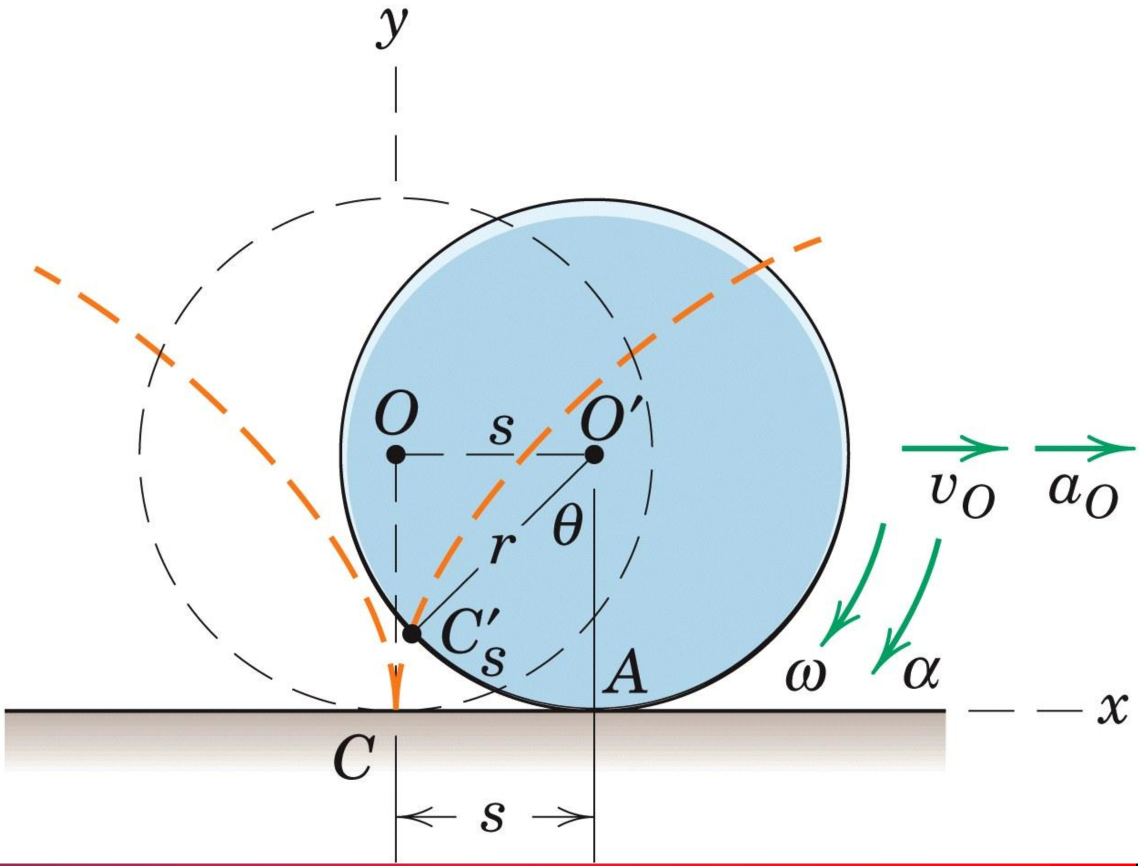
Absolute Motion

Rolling Wheel (no slip)

$$s = r\theta$$

$$v_o = \dot{s} = r\dot{\theta} = r\omega$$

$$a_o = \dot{v}_o = \ddot{s} = r\ddot{\theta} = r\dot{\omega} = r\alpha$$



Rolling Wheel (no slip)

$$x = s - r\sin\theta = r(\theta - \sin\theta) \qquad y = r - r\cos\theta = r(1-\cos\theta)$$

$$\dot{x} = r\dot{\theta}(1-\cos\theta) = v_o(1-\cos\theta) \qquad \dot{y} = r\dot{\theta}\sin\theta = v_o\sin\theta$$

$$\ddot{x} = \dot{v}_o(1-\cos\theta) + v_o\dot{\theta}\sin\theta \qquad \ddot{y} = \dot{v}_o\sin\theta + v_o\dot{\theta}\cos\theta$$

$$\ddot{x} = a_o(1-\cos\theta) + r\omega^2\sin\theta \qquad \ddot{y} = a_o\sin\theta + r\omega^2\cos\theta$$

For θ=0

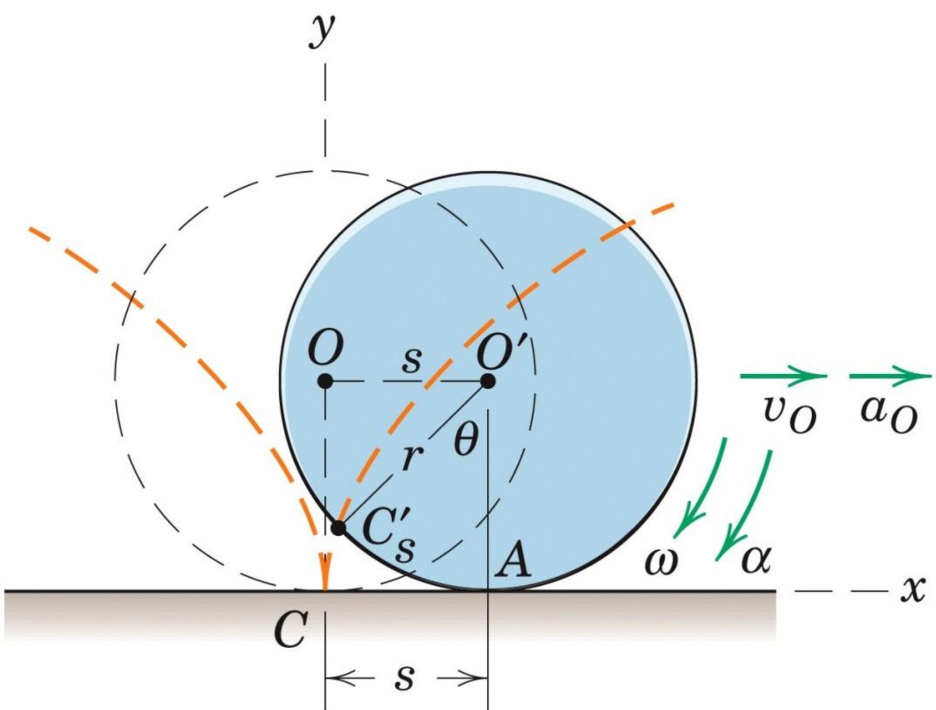
$$\ddot{x} = 0$$

$$\ddot{y} = r\omega^2$$

For any θ

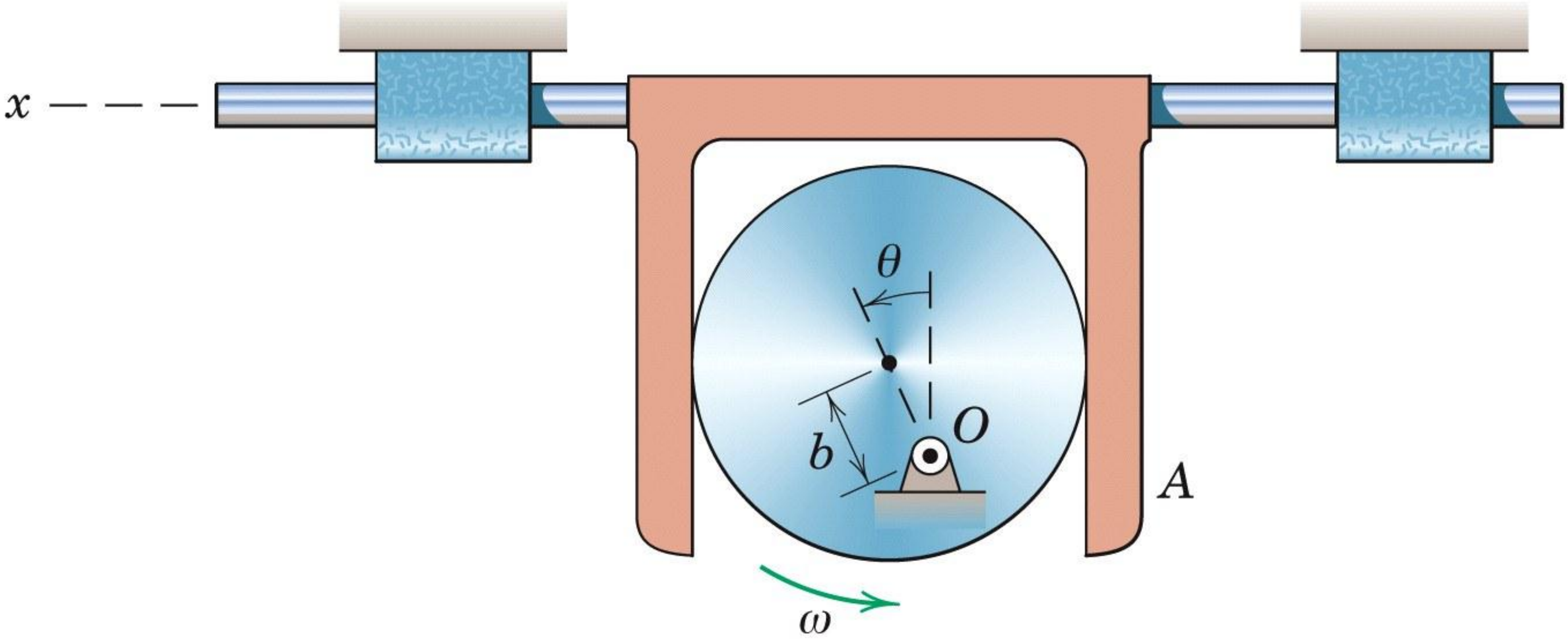
$$\vec{v} = \dot{x}\vec{i} + \dot{y}\vec{j}$$

$$\vec{a} = \ddot{x}\vec{i} + \ddot{y}\vec{j}$$



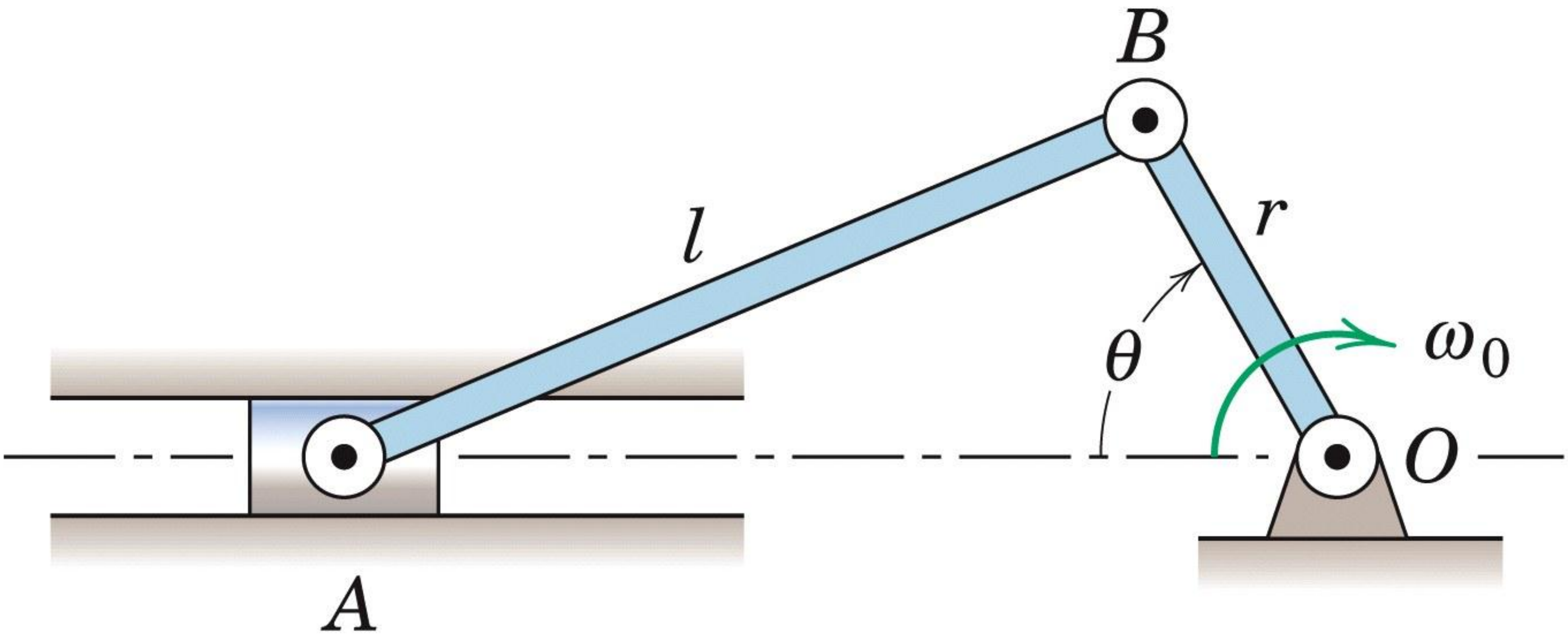
Problem 5/43

Rotation of the lever OA is controlled by the motion of the contacting circular disk whose center is given a horizontal velocity v. Determine the expression for the angular velocity ω of the lever OA in terms of x.



Problem 5/57

One of the most common mechanisms is the slider-crank. Express the angular velocity ω_{AB} and angular acceleration α_{AB} of the connecting rod AB in terms of the crank angle θ for a given constant crank speed ω_0. Take ω_{AB} and α_{AB} to be positive counterclockwise.



Relative Velocity

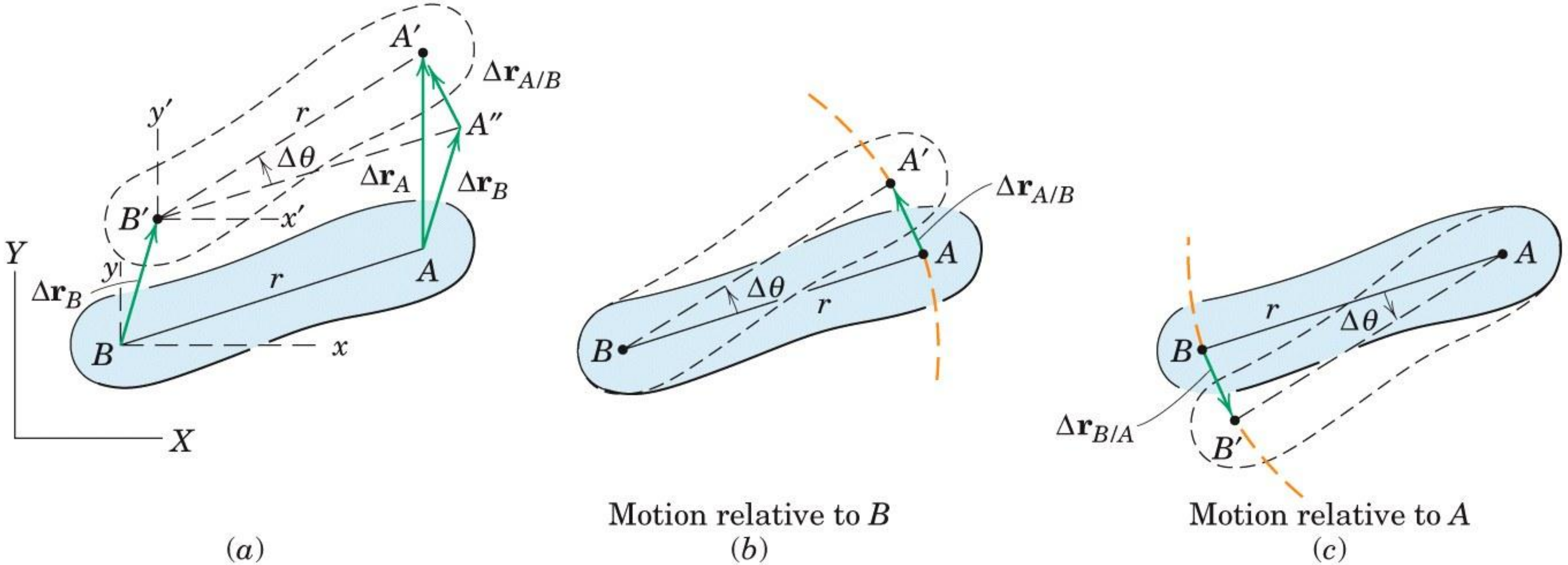
Relative Velocity Due to Rotation

$$\Delta\vec{r}_A = \Delta\vec{r}_B + \Delta\vec{r}_{A/B}$$

$$\boxed{\vec{v}_A = \vec{v}_B + \vec{v}_{A/B}}$$

$$\boxed{v_{A/B} = r\omega}$$

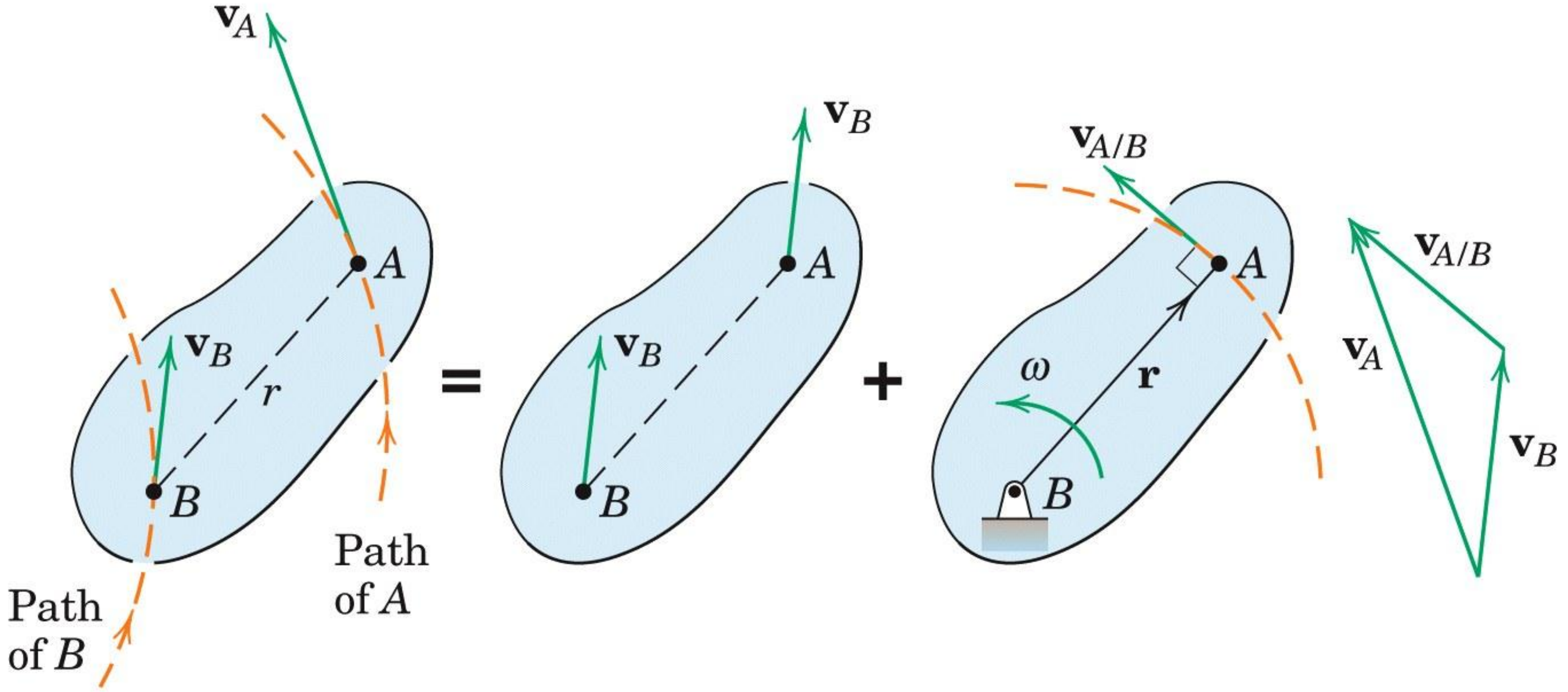
$$\boxed{\vec{v}_{A/B} = \vec{\omega} \times \vec{r}}$$



Relative Velocity

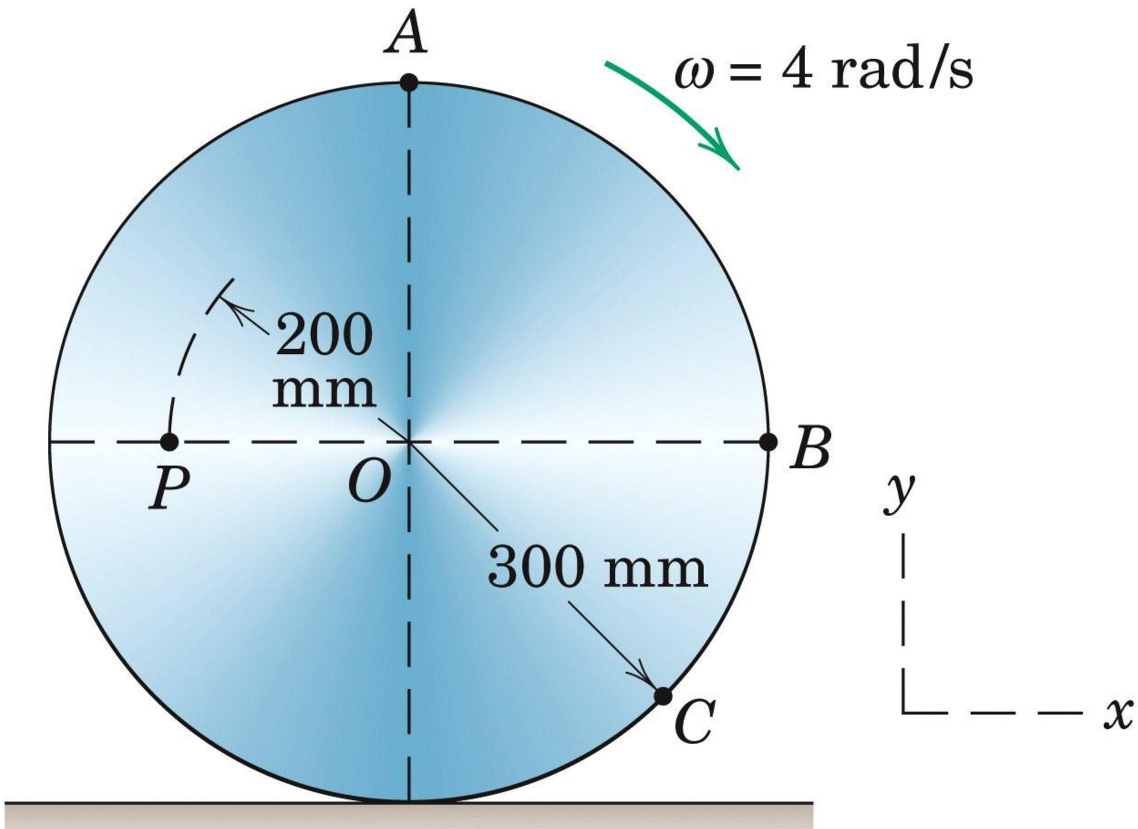
Interpretation of Relative-Velocity Equation

Velocity of A = vector sum of translational portion $\vec{v}_B$ + *rotational portion* $\vec{v}_{A/B} = \vec{\omega} \times \vec{r}$



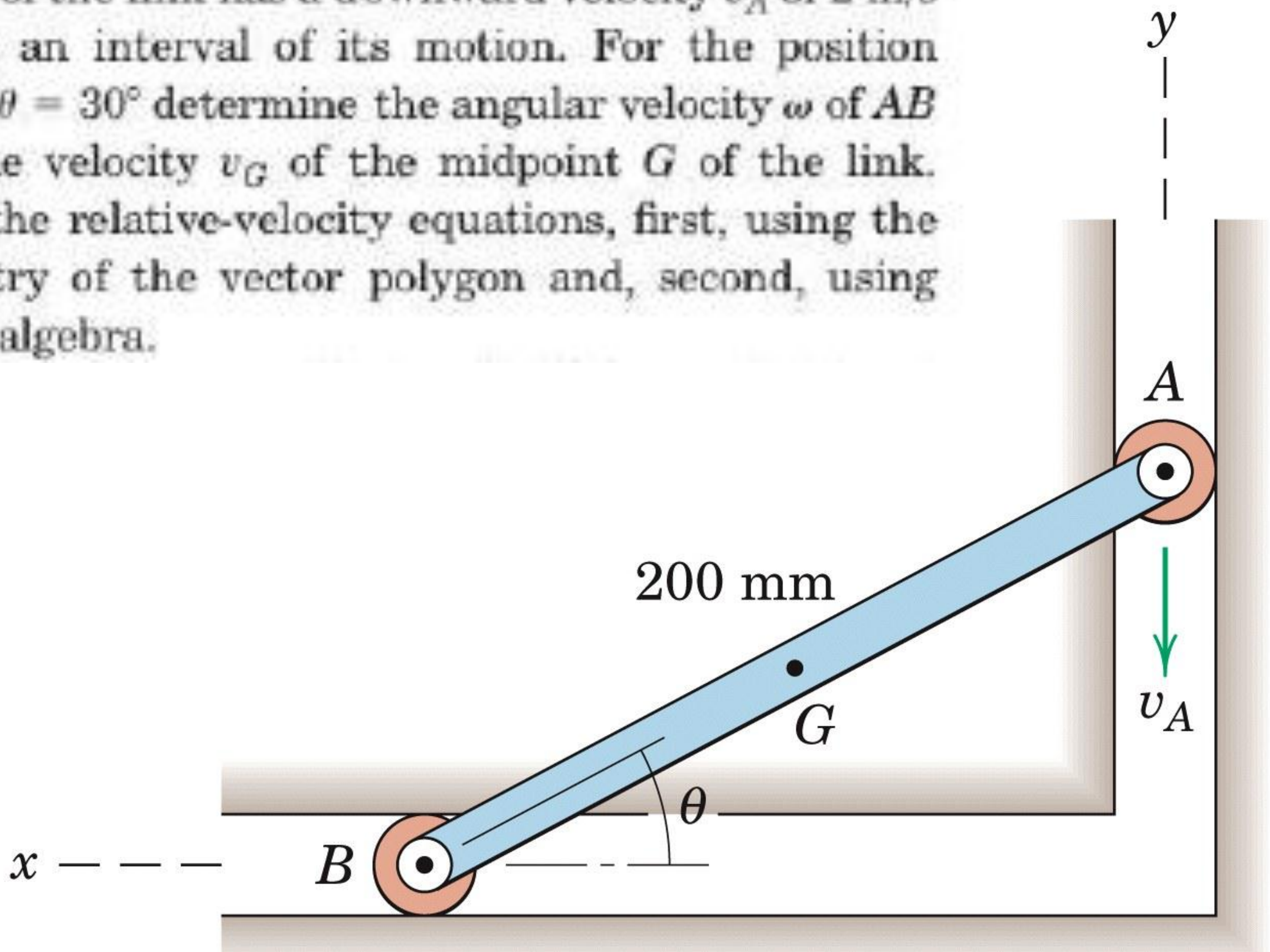
Problem 5/72

The circular disk rolls without slipping with a clockwise angular velocity $\omega = 4$ rad/s. For the instant represented, write the vector expressions for the velocity of A with respect to B and for the velocity of P.



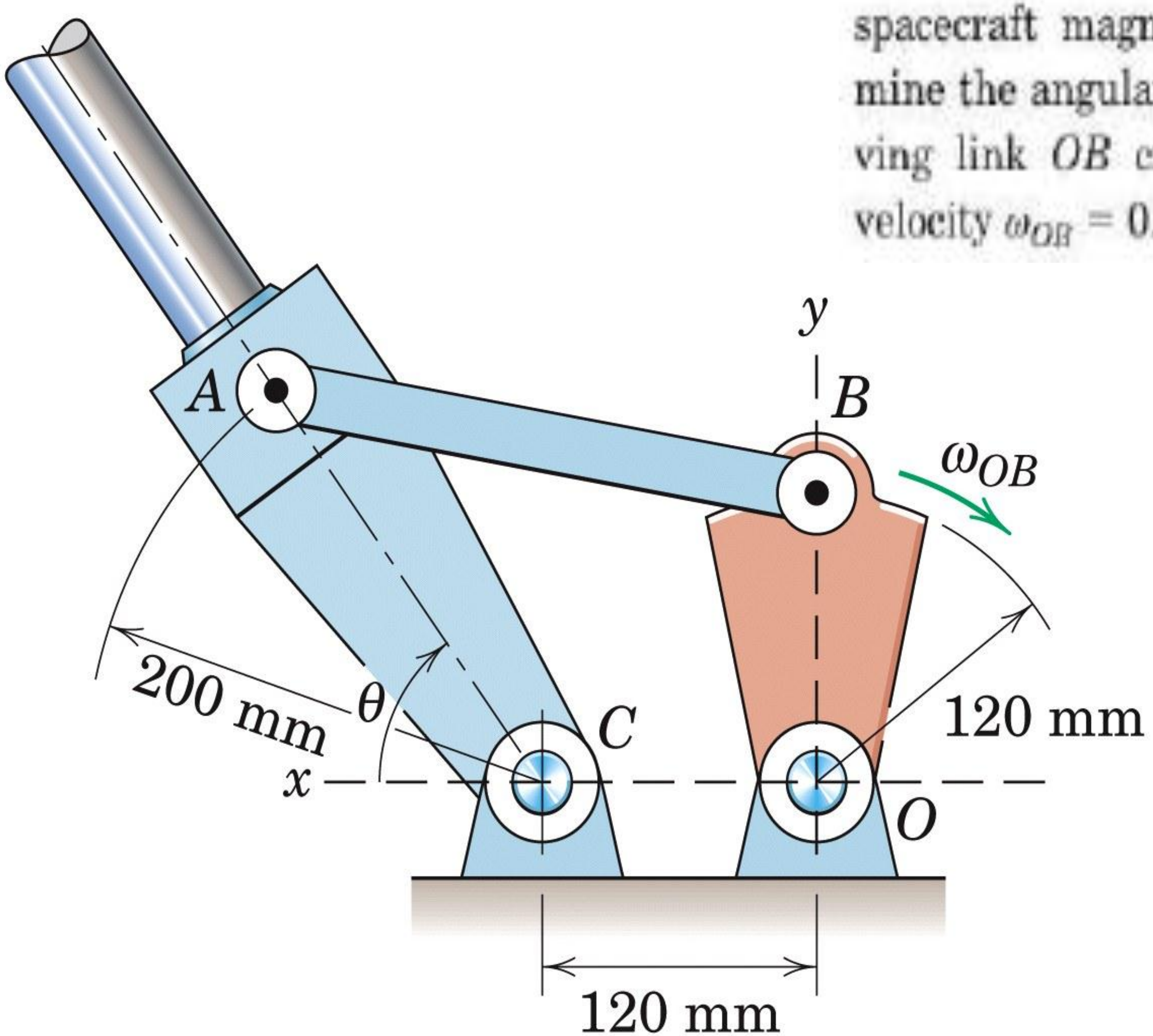
Problem 5/82

End A of the link has a downward velocity v_A of 2 m/s during an interval of its motion. For the position where $\theta = 30°$ determine the angular velocity ω of AB and the velocity v_G of the midpoint G of the link. Solve the relative-velocity equations, first, using the geometry of the vector polygon and, second, using vector algebra.



Problem 5/89

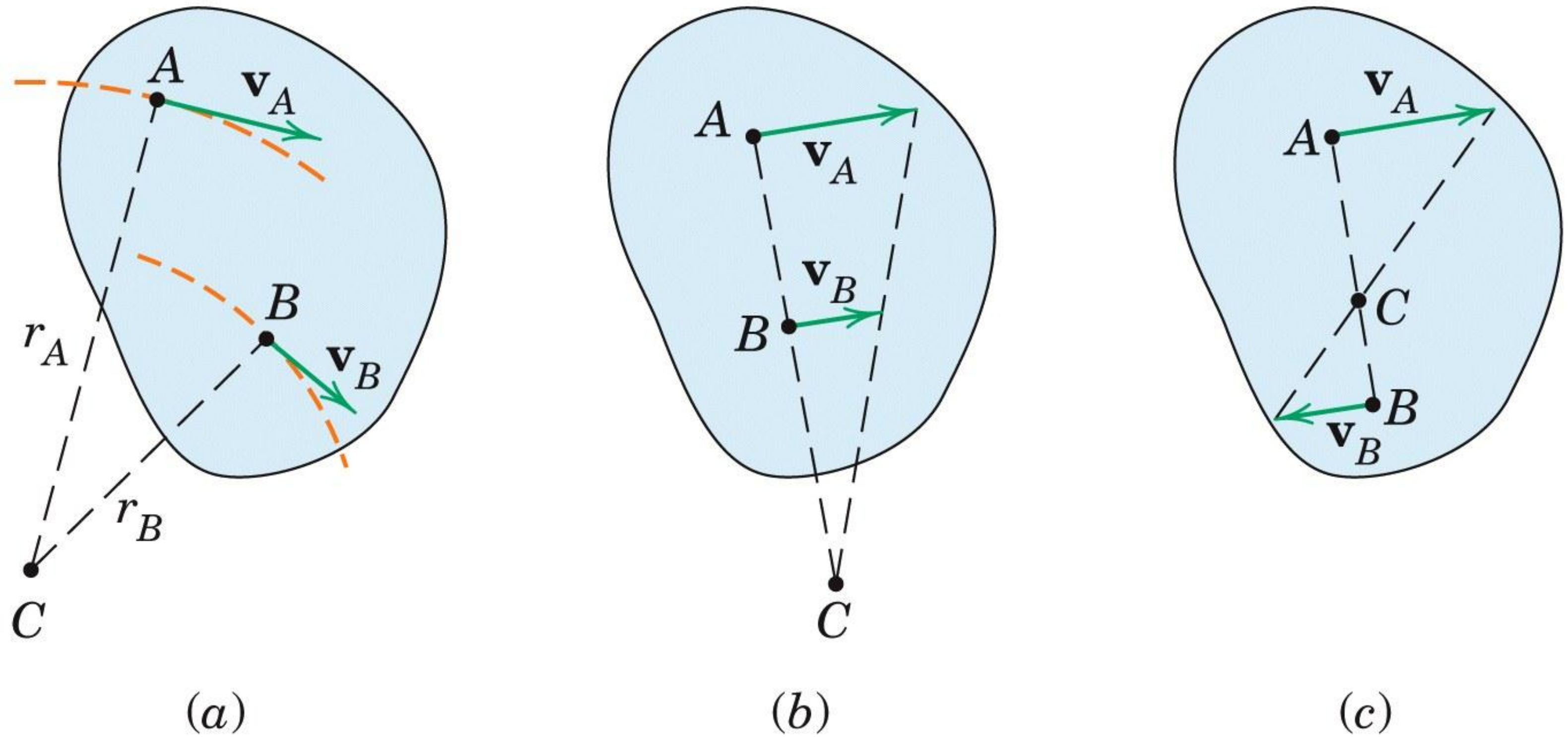
The elements of the mechanism for deployment of a spacecraft magnetometer boom are shown. Determine the angular velocity of the boom when the driving link OB crosses the y-axis with an angular velocity $\omega_{OB} = 0.5$ rad/s if $\tan\theta = 4/3$ at this instant.



Instantaneous Center of Zero Velocity

Locating the Instantaneous Center

***Instantaneous center of zero velocity* (C)** : For a body with plane motion , the unique point on the body such that, instantaneously, $\vec{v_c} = \vec{0}$.
Instantaneous axis of zero velocity : The axis which is normal to the plane of motion and passes through C. As far as the velocities are concerned, the body can be considered to be executing pure rotation, instantaneously , around the instantaneous axis of zero velocity.



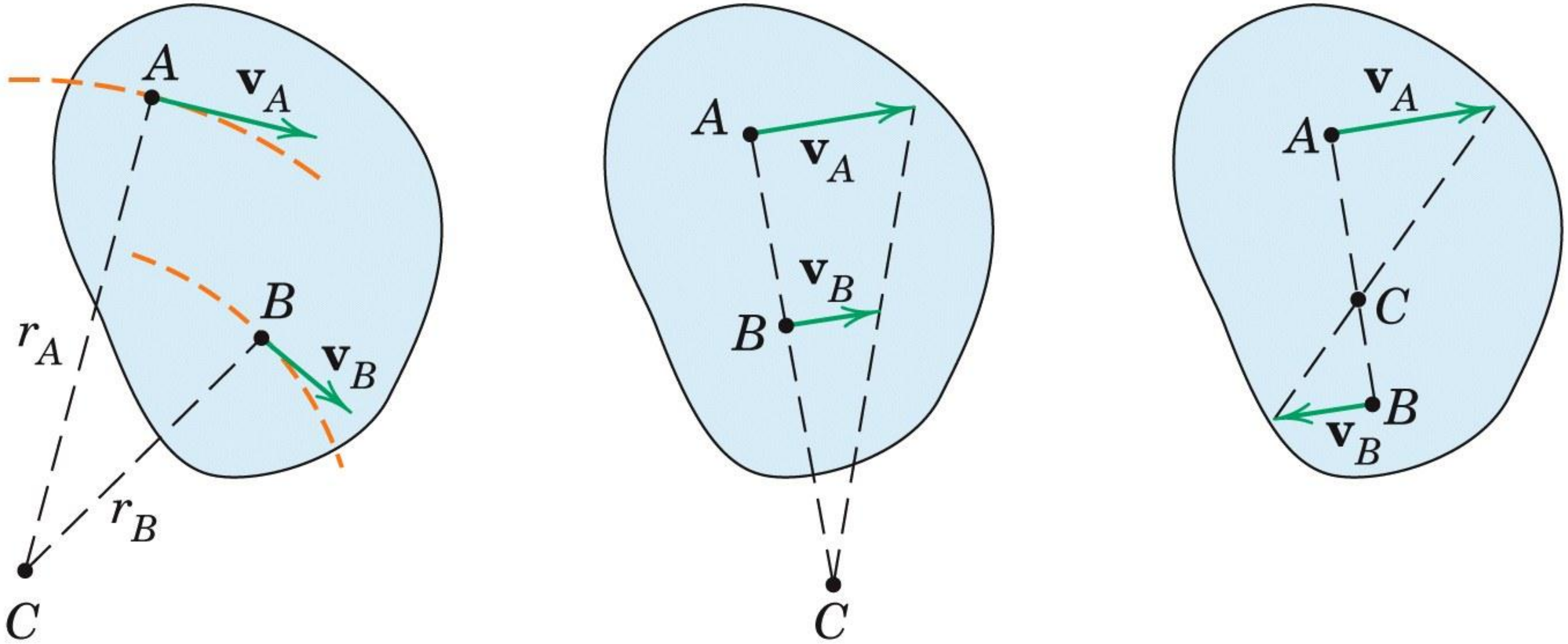
Instantaneous Center of Zero Velocity

Locating the Instantaneous Center

$v_A = \omega r_A$

$v_B = \omega r_B$

- $\vec{\omega}$ and C can be determined if $\vec{v}_A$ and direction of $\vec{v}_B$ (or, $\vec{v}_B$ and direction of $\vec{v}_A$) known.
- If $\vec{\omega}$ and C known, velocity of any point can be determined.

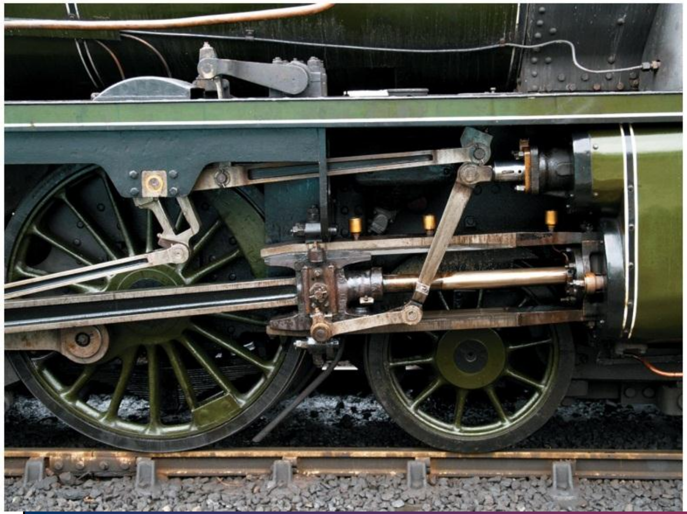


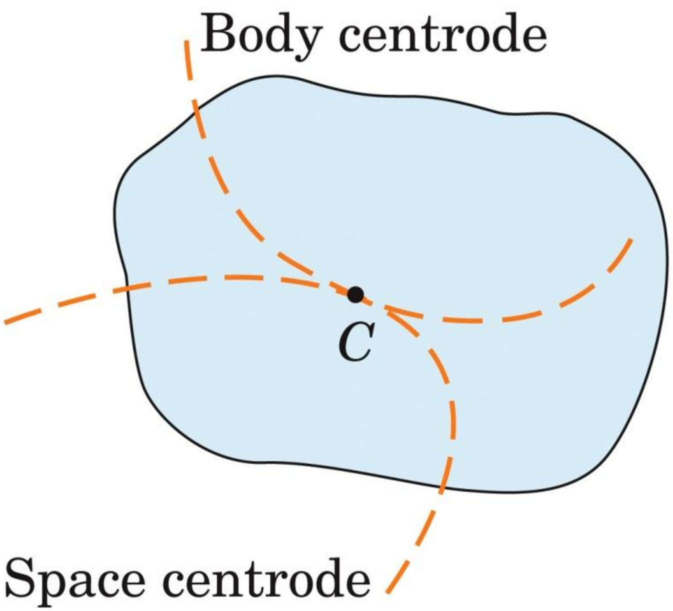
(a) (b) (c)

Instantaneous Center of Zero Velocity

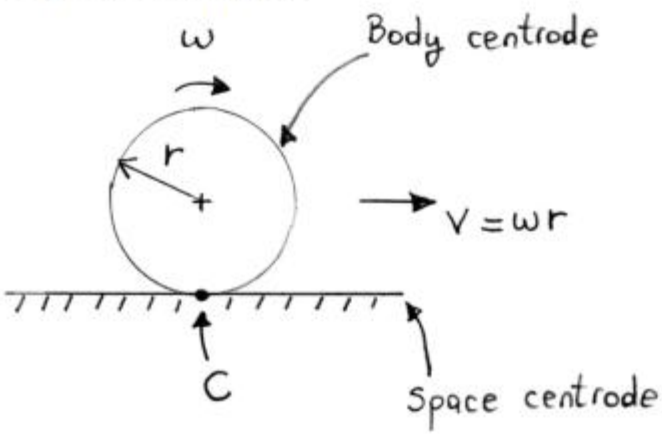
Motion of the Instantaneous Center

The position of C changes as the position of the body changes.
Space centrode : Locus of C on the fixed (i.e., nonmoving) body.
Body centrode : Locus of C on the body itself.
Body centrode rolls, without slippage, on the space centrode during motion (C is contact point of two curves).



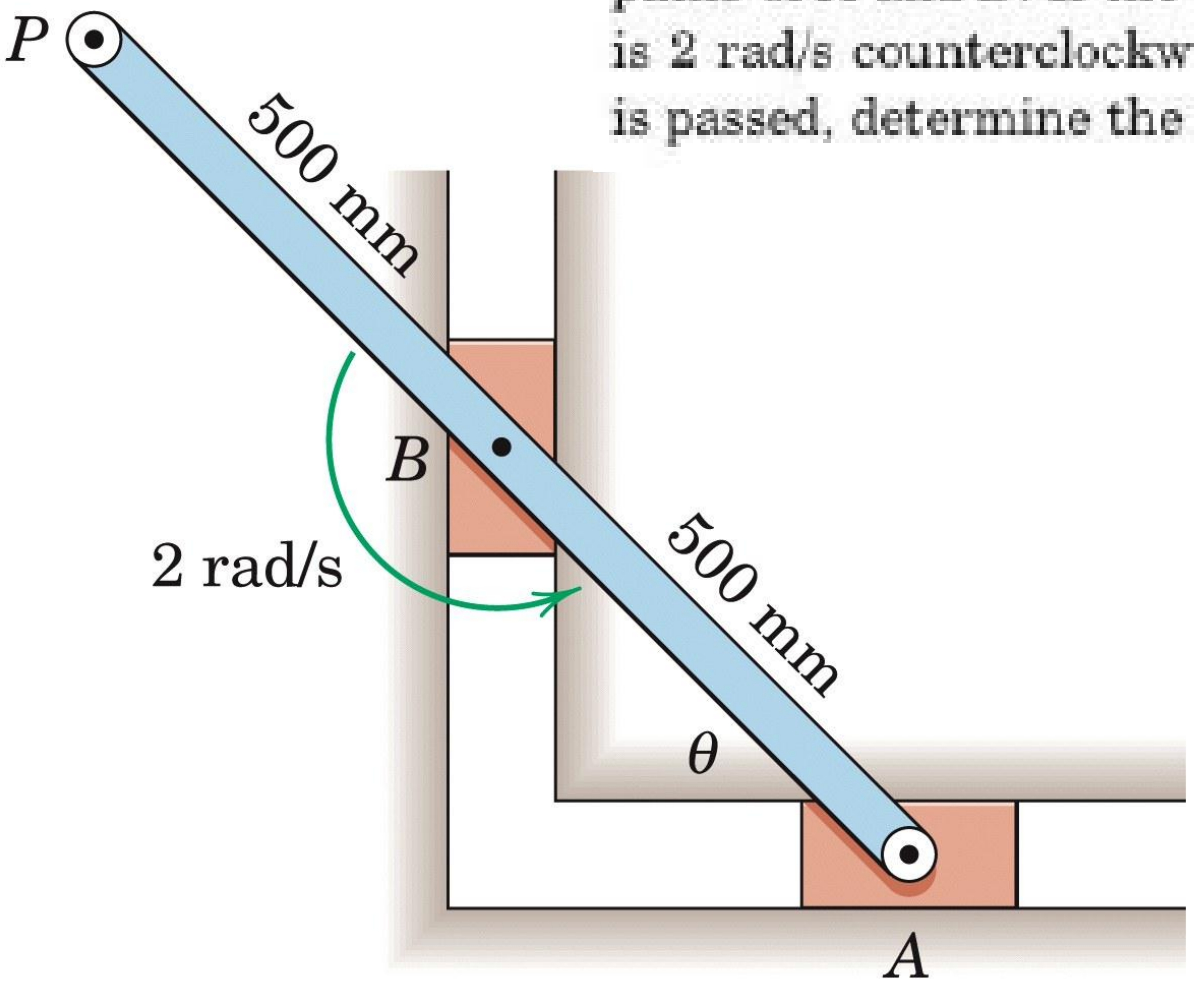


Disk rolling without slippage



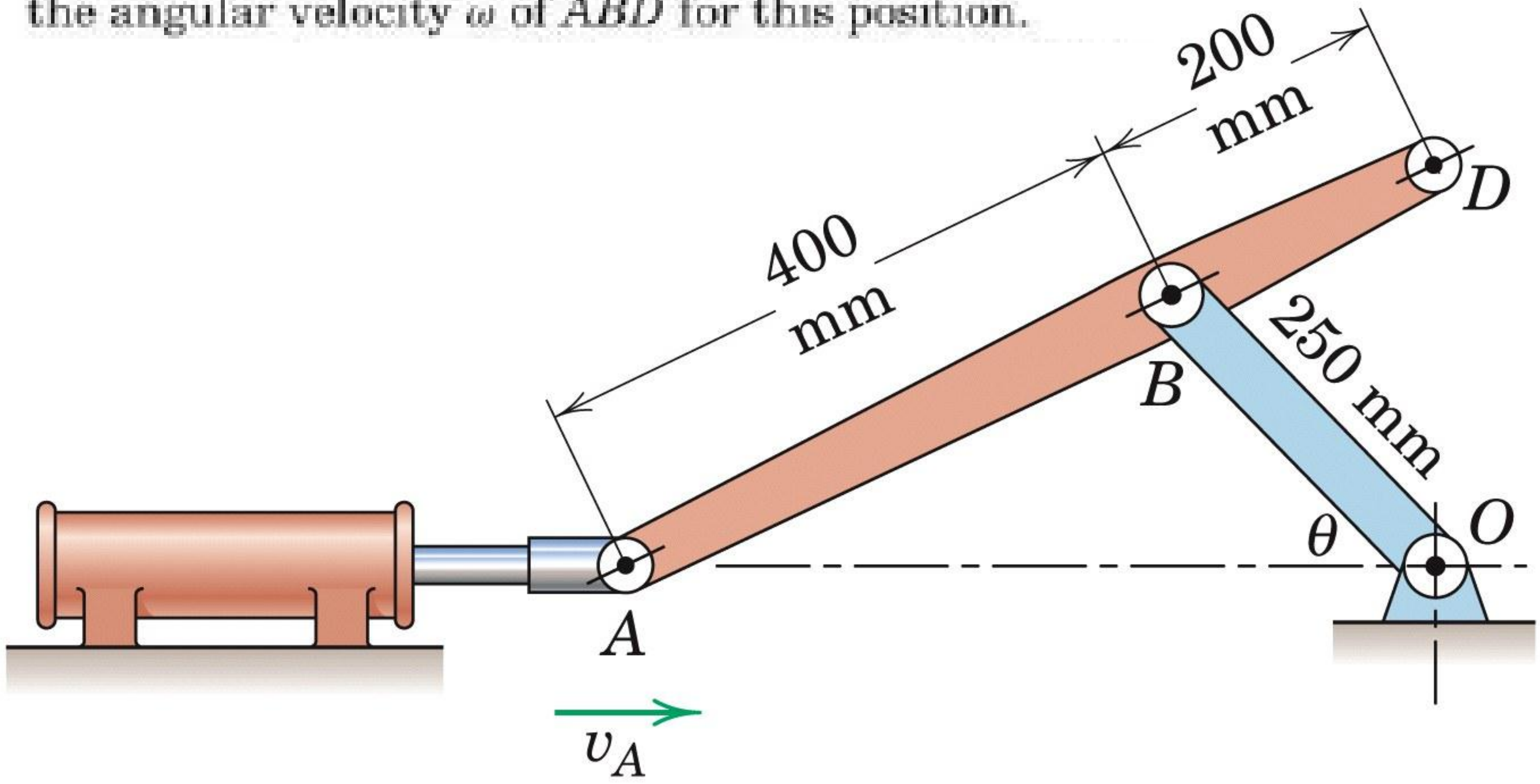
Problem 5/100

Motion of the bar is controlled by the constrained paths of A and B. If the angular velocity of the bar is 2 rad/s counterclockwise as the position $\theta = 45°$ is passed, determine the speeds of points A and P.

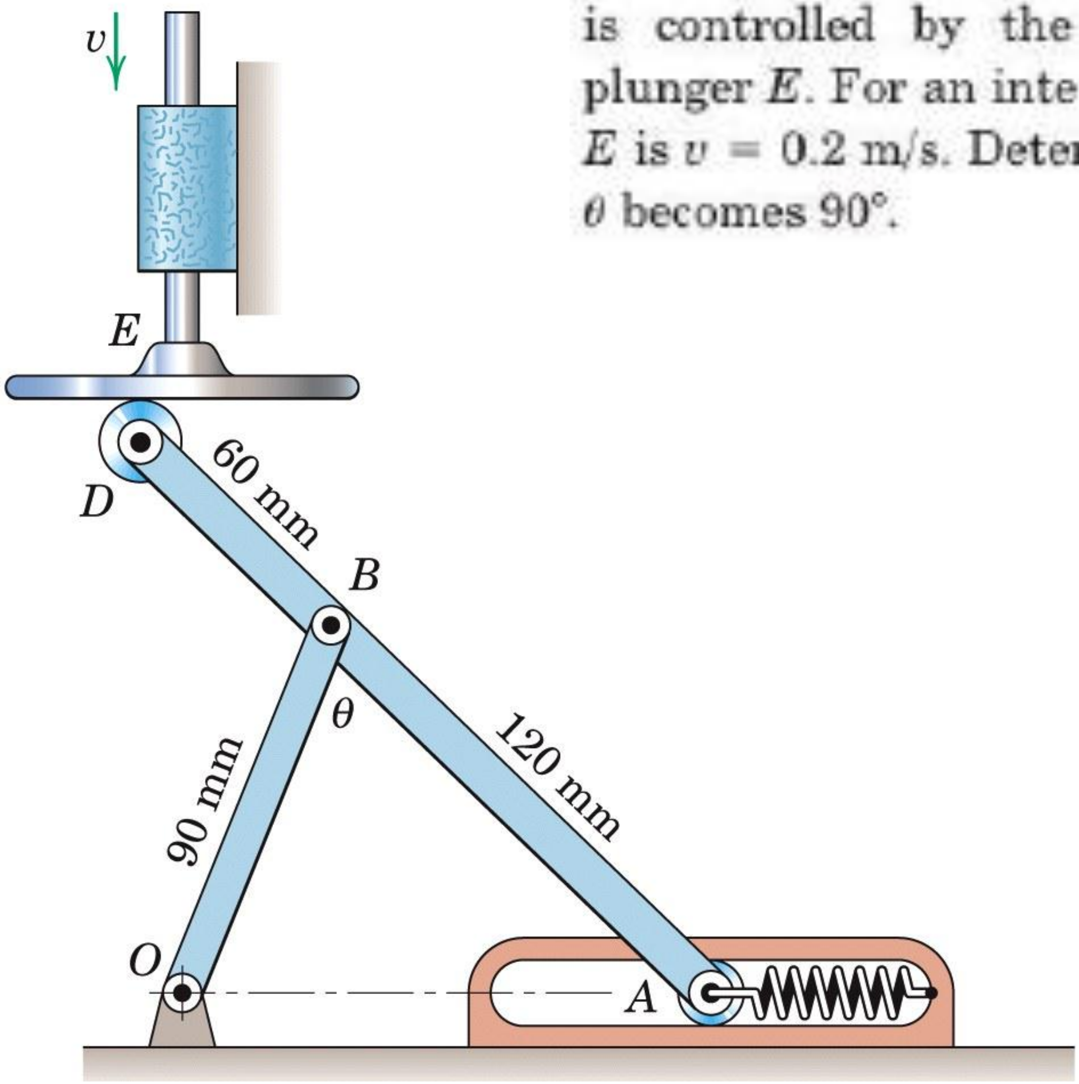


Problem 5/114

The hydraulic cylinder produces a limited horizontal motion of point A. If $v_A = 4$ m/s when $\theta = 45°$, determine the magnitude of the velocity of D and the angular velocity ω of ABD for this position.



Problem 5/116



Motion of the roller A against its restraining spring is controlled by the downward motion of the plunger E. For an interval of motion the velocity of E is $v = 0.2$ m/s. Determine the velocity of A when θ becomes 90°.

Homework 4 (Chapter 5)

Problems for Week 09

- Problem 5/53
- Problem 5/56
- Problem 5/75
- Problem 5/84
- Problem 5/108
- Problem 5/120

- There will be additional problems for the next week(s).
- Solve each weeks problems in that week, do not wait for the last day!
- Submit the Homework 04 in due time.
- You will have a quiz in the 1st hour of Week 11. A problem will be asked from the Homework 4 questions.
- Your homework grade will be assessed only from that question of your homework.

Due date: 1st hour of Week 11

English – Turkish Dictionary

particle	Maddesel nokta, parçacık	**kinematics**	Kinematik, hareket bilimi	**resultant**	bileşke
kinetics	Kinetk, hızbilim	**Angular rate**	Açısal hız	**displacement**	Yer değiştirme
velocity	hız	**acceleration**	ivme	**revolution**	devir
translation	öteleme	**rotation**	dönme	**rod**	çubuk
relative	Bağıl, göreceli	**rectilinear**	doğrusal	**curvilinear**	eğrisel
position	konum	**displacement**	Yer değiştirme	**component**	bileşen
plane	düzlem	**normal**	dik	**instantaneous**	anlık
tangential	teğetsel	**slider**	piston	**Connecting rod**	Piston kolu
lever	levye	**deflection**	sapma	**swinging**	salınan
circular	dairesel	**İnitial velocity**	İlk hız	**reciprocating**	karşılıklı
rolling	yuvarlanan	**polar**	kutupsal	**pendulum**	sarkaç
horizontal	yatay	**vertical**	düşey	**absolute**	mutlak
link	Bağlantı, biyel	**crank**	Ana mili	**rigid**	Katı, şekil değiştirmeyen